The Calculation of Recoverable Reserves

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Abstract

Resource/reserve estimates are critical to any mining project. Mine productivity and equipment selectivity must be balanced and considered in the estimation of recoverable reserves. This has been a long-standing challenge in mining geostatistics stemming from the early days of the discipline. Many techniques have been developed and others have evolved to address this problem.

This paper reviews conventional and emerging approaches to resource/reserve estimation. Specifically, change of support models, kriging, uniform conditioning and simulation are considered. These different approaches are then illustrated using data from a gold deposit, and compared on the basis of a grade tonnage curve. This comparison revealed that the indirect lognormal, discrete Gaussian model, uniform conditioning with kriging and simulation yielded reasonable and comparative results. The use of kriging and the affine correction performed poorly relative to the other methods. Guidelines for consideration and implementation of these methods are provided.

1 Introduction

Recoverable reserves/resources are integral to all mineral projects. Different types of estimates are required at different stages of the mine life: global reserves are needed at the project development stage, while local reserves are required for detailed mine planning and pit optimization. While the estimation of resources is the same prior to the calculation of reserves, the latter requires economic evaluation and pit/underground planning. Further, this estimation is made more complex when one considers the underlying issues related to equipment and the corresponding scale of production.

There is no doubt that recoverable reserves are a function of the selectivity that can be achieved by the equipment being used. Large equipment is more productive than small equipment, but this comes at the price of selectivity. Consequently, larger mining equipment cannot distinguish ore and waste as easily or accurately as small mining equipment. The selective mining unit (SMU) is *the smallest volume of material on which ore waste classification is determined* [10]; hence it is a clear measure of the equipment selectivity and has a large impact on the reserve calculation (see [4] for more information).

While SMU's are necessarily the smallest volumes for production, reserves are calculated for larger mine planning blocks, typically referred to as panels. Calculation of panel grades,
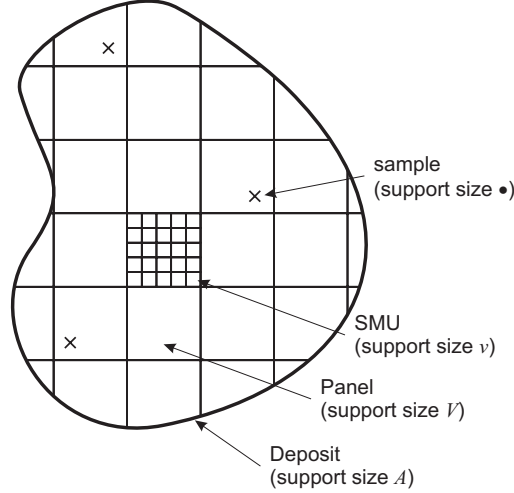
Figure 1: Typical setting for reserve calculation: samples are represented by the X's; the SMU's denote the small blocks, v; the panels as represented by the larger blocks, V; and the deposit is considered as the entire area, A..

tonnages, ore/waste classification, etc. are based on the constituent SMU's within the panel. The panel reserves can be further upscaled to provide an estimate of the reserves for the entire deposit. Figure 1 shows a schematic illustration of the relation between these scales of production, planning and reporting.

Achieving the balance between productivity and selectivity is certainly a challenge, yet the effect of dilution due to scale of production presents another issue for consideration. Consider assays from an exploration drilling program. There will typically be samples with very low grades and samples with very high grades; these extreme values contribute to a histogram with high variance. If the samples are composited to a larger scale, higher grade material will be averaged with lower grade material. The effect of this averaging leads to a histogram of composited data that has a lower variance than the assays (see Figure 2). Clearly, the distribution of grades within the deposit changes as the size of the sample, or block, increases.

Given these complex, inter-related issues, it is easy to see why so many different resource estimation methods have been developed for different deposit types and settings. Incorporating scale effects in the resource estimate is critical. There are analytical change of support models to correct the point scale histogram to one that approximates the SMU scale histogram. Global resources can then be calculated directly from the upscaled histogram. Local resources can be estimated through linear, non-linear, and simulation based algorithms. The local resources can be upscaled to get global resources.

This paper will compare some of the more common reserve estimation techniques on the basis of (i) applicability, (ii) amount of time and effort required for implementation, and (iii) accuracy of the results. A brief review of volume variance and resource calculation is first presented; this is followed by a distinction between global and local resource estimation methods. A small comparative example is also included.
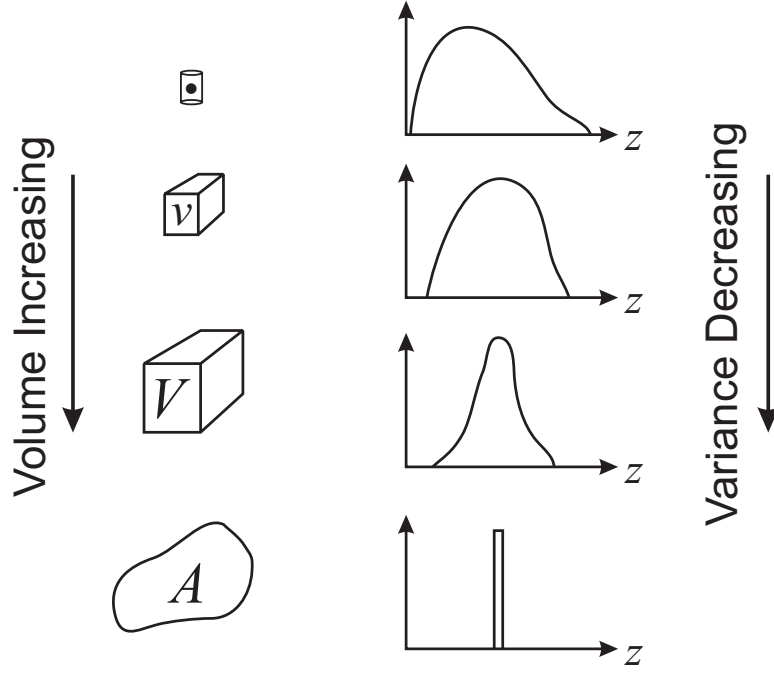
Figure 2: Change in the histogram with a change in the support size (redrawn from Deutsch, 2001).

2   Background

The scale issue remains a long standing challenge in geostatistics; certainly the mining industry has struggled with its complexity since the pioneering work of Krige and Sichel in the 1950s [5, 9]. Since that time, many researchers have worked to quantify the affect of volume on the spatial variability of the phenomena, and how this translates to an effect on the variability in the grades distribution.

2.1   Scale Effect on Variability

Gammabar.   The average variogram or "gammabar", $\overline{\gamma(V,v)}$, is a widely used measure to estimate the variability of any arbitrary volume. Gammabar values represent the mean value of $\gamma(\mathbf{u})$ when the head of the variogram describes the block $V(\mathbf{u})$ and the tail describes the block $v(\mathbf{u})$ where $\mathbf{u}$ is a location vector (see Figure 3). The blocks v and V may or may not be the same size, they may be separated by some distance, and/or they may be irregularly shaped.

Gammabar values can be calculated exactly with the following integral:

$$\overline{\gamma(V,v)} = \frac{1}{V \cdot v} \int_{V(\mathbf{u})} dx \int_{v(\mathbf{u}')} \gamma(x - x') \, dx' \tag{1}$$

Alternatively, the gammabar can be numerically approximated by a sextuple summation (3 directions for each volume) taken over the discretization of the two volumes. This is
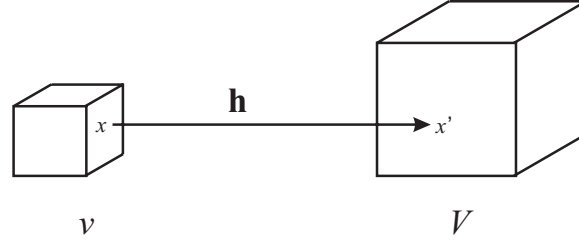
Figure 3: Schematic illustration of two arbitrary volumes, v and V, separated by a lag distance, **h**.

simple, relatively fast and works for any variogram type and size/shape of the volumes. This flexibility makes the numerical approach the common modern approach.

Consider the case of calculating the gammabar of a volume with itself. If the volume v is very small, say a point support, the $\overline{\gamma(v,v)}$ will be equal to zero since the separation vector, **h**, will equal zero. In contrast, consider now a much larger volume, say the domain A, discretization of this volume into many points for numerical integration will involve averaging the variogram of points separated by very large separation distances, **h**; this results in a $\overline{\gamma(A,A)}$ equal to the variance of the field. Thus as the volume increases, the gammabar values will also increase and tend towards the variance of the field.

Dispersion Variance. The dispersion variance, $D^2(v,V)$, is a measure of the variability of the small volume v within a larger volume V. The additivity of variance property was used to establish Krige's relation [2, 3]:

$$D^2(v,A) = D^2(v,V) + D^2(V,A) \tag{2}$$

Therefore, the variance of the small blocks v in the area A is equal to the variance of the small blocks v within the big blocks V plus the variance of the big blocks V in the area A, that is $v \subset V \subset A$. This equation can be rearranged to solve for any of the terms.

Dispersion variances can be estimated in one of two ways: (1) use the available data and estimate the block variance by upscaling the data directly, and (2) estimate the block variance using the fitted variogram model. Ideally, both methods would be used and any differences reconciled. However, there is usually insufficient data to perform method 1. As a result, method 2 has become the accepted default practice for estimating block variances.

Any dispersion variance can be estimated using the gammabar approach. Each dispersion variance calculation requires two gammabar values: one for the large volume and one for the small volume. The dispersion variance is [3]:

$$D^2(v,V) = \overline{\gamma(V,V)} - \overline{\gamma(v,v)} \tag{3}$$

Note that any dispersion variance estimated with the variogram is sensitive to the nugget effect. A sensitivity study should be performed when there is a large amount of uncertainty in the modeled variogram parameters.

2.2   Recoverable Reserve Calculation

We have already seen how a change in volume affects the spatial variability of the grades (as characterized by the variogram), and the corresponding affect on the grades distribution. Of course, a change in the grades distribution must necessarily impact the recoverable reserves calculation. This section is dedicated to the different approaches for reserves calculation.

A mining operation can never recover all of the in-situ resources. Some portion of the deposit will be inaccessible or unprofitable to mine. Recoverable reserves are a subset of the resources that can be economically and technically mined.

Assume that we only want to select material with a grade above a specified minimum grade, or cutoff grade, z_c. The cutoff grade can be defined based on any number of considerations. A simplistic formula that can be used is:

$$z_c = \frac{c_t + (c_o - c_w)}{p \cdot r} \tag{4}$$

where c_t is the cost of milling, c_o is the cost of mining ore, c_w is the cost of mining waste, p is the metal selling price, and r is the fraction of metal recovered during processing. The deposit is classified into ore and waste using the cutoff grade.

Histogram Based.   Recoverable reserves can be estimated directly from a histogram. The tonnage, quantity of metal, and average grade of the ore can be calculated. The area that the histogram represents and the support volume for the histogram impact the reserves.

Let T_0 be the total tonnage of the deposit. The recovered tonnage is:

$$T_A(z_c) = T_0 \cdot [1 - F_Z(z_c)] = T_0 \cdot \int_{z_c}^{+\infty} f_z(z)dz \tag{5}$$

where $f_z(z)$ is the probability density function or frequency distribution of the grades. The quantity of metal can then be calculated as

$$Q_A(z_c) = T_0 \cdot \int_{z_c}^{+\infty} z f_z(z)dz \tag{6}$$

and the mean grade of the recovered material is given by

$$M_A(z_c) = \frac{Q_A(z_c)}{T_A(z_c)} \tag{7}$$

The impact of the selection volume can be understood by performing this analysis for different selection volumes. The expected result is that as the selection volume increases, the number of tonnes will increase and the average grade will decrease.

The spatial continuity of the variable also impacts the calculated reserves through the dispersion variance. Reserves for continuous variables will not change substantially as the selection volume increases; whereas, reserves for variables that are not continuous will change drastically as the support volume increases.

Only global reserves can be calculated using the histogram based approach. It provides an estimate of the reserves for a given volume where the data is representative.

Block Based. Let us assume that we only want to select SMU's with a grade above a specified minimum grade, or cutoff grade, z_c. This leads to the definition of an indicator function that classifies the SMU as ore or waste according to:

$$i_v\left(\mathbf{u}_\beta; z_c\right) = \begin{cases} 1 & if\ z_v\left(\mathbf{u}_\beta\right) > z_c \\ 0 & otherwise \end{cases} \tag{8}$$

where $\mathbf{u}_\beta$ refers to the SMU centered at location $\mathbf{u}_\beta$, $z_v\left(\mathbf{u}_\beta\right)$ is the grade of the SMU, and z_c is the cutoff grade. The tonnes of ore above the cutoff grade for any panel can be calculated using the indicator function and the SMU tonnage:

$$T_V\left(\mathbf{u}_\alpha; z_c\right) = \sum_{\beta=1}^{N_v} i_v\left(\mathbf{u}_\beta; z_c\right) t_v(\mathbf{u}_\beta) \quad \mathbf{u}_\beta \in \mathbf{u}_\alpha \tag{9}$$

where $T_V\left(\mathbf{u}_\alpha; z_c\right)$ is the tonnage of ore above the cutoff grade for panel V centered at location $\mathbf{u}_\alpha$, $t_v(\mathbf{u}_\beta)$ is the tonnage of the SMU at $\mathbf{u}_\beta$, and $\beta = 1, \ldots, N_v$ where N_v is the number of SMU's contained within the panel, V. The quantity of metal above the cutoff grade is:

$$Q_V\left(\mathbf{u}_\alpha; z_c\right) = \sum_{\beta=1}^{N_v} i_v\left(\mathbf{u}_\beta; z_c\right) \cdot t_v(\mathbf{u}_\beta) \cdot z_v\left(\mathbf{u}_\beta\right) \quad \mathbf{u}_\beta \in \mathbf{u}_\alpha \tag{10}$$

The mean grade of the ore is given by:

$$M_V\left(\mathbf{u}_\alpha; z_c\right) = \frac{Q_V(\mathbf{u}_\alpha; z_c)}{T_V(\mathbf{u}_\alpha; z_c)} \tag{11}$$

The resource estimate for each panel can be upscaled for a global resource estimate.

$$T_A\left(z_c\right) = \sum_{\alpha=1}^{N_V} T_V(\mathbf{u}_\alpha; z_c) \quad \mathbf{u}_\alpha \in A$$

$$Q_A\left(z_c\right) = \sum_{\alpha=1}^{N_V} Q_V(\mathbf{u}_\alpha; z_c) \quad \mathbf{u}_\alpha \in A$$

$$M_A\left(z_c\right) = \frac{Q_A(z_c)}{T_A(z_c)} S$$

where $\alpha = 1, \ldots, N_V$ and N_V is the number of panels within the deposit, A.

Recoverable reserves calculated from a block model should be similar to the reserves calculated from a histogram. The block based reserves will take more time to produce, but more can be done with the block model; i.e. mine planning and selective reserves within a constrained area.

3 Global Reserve Estimation

Global reserves are used for disclosure and financial purposes. At this stage, the location of the reserves within the deposit is not critical. Ensuring that the total recoverable reserves are accurate is the most important criteria at this stage.

It is common practice to estimate recoverable resources directly from the grade histogram [2]. However, the histogram of the sample data is not representative of the blocks that will be mined. A correction needs to be applied to the histogram to account for the larger blocks that will actually be mined. Many case studies have been published showing that very good results can be achieved with this method [8].

Recall that the variance of blocks within the deposit can be estimated using dispersion variance theory. Change of support models use this block variance to alter the sample histogram so it represents the blocks that will actually be mined. There are several common change of support models: (i) affine, (ii) indirect log-normal, and (iii) discrete Gaussian model. The different change of support models are applied in similar manners. However, each change of support model makes different assumptions regarding the point to block bivariate relationship and the corresponding shape change of the histogram.

All change of support models make two common assumptions: (1) the variance decreases as the volume increases, and (2) the mean does not change from one scale to the next. Each model assumes a different shape change as the volume increases. This is an important consideration when choosing the change of support model to use.

3.1 Volume Variance Relations

As the block size increases, the variance decreases. We can define a change of support parameter, or variance reduction factor f, for use in the change of support calculation. Let us define the reduction factor f as the variance of the larger volume over the variance of the smaller volume:

$$f = \frac{D^2(v, A)}{D^2(\cdot, A)} \tag{12}$$

Recalling the additivity of dispersion variances of Equation (2), we can solve for $D^2(v, A)$:

$$D^2(v, A) = D^2(\cdot, A) - D^2(\cdot, v) \tag{13}$$

Substitution of this into Equation (12) reveals

$$f = \frac{D^2(\cdot, A) - D^2(\cdot, v)}{D^2(\cdot, A)} \tag{14}$$

$$= 1 - \frac{D^2(\cdot, v)}{D^2(\cdot, A)} \tag{15}$$

Recall that $D^2(\cdot, A)$ is the variance of the points in the deposit A, or σ^2, and that $D^2(\cdot, v)$ is the variance of the points within the blocks v, or $\overline{\gamma}(v, v)$:

$$f = 1 - \frac{\overline{\gamma}(v, v)}{\sigma^2} \tag{16}$$

Therefore, f is a function of the modeled variogram and the variance of the points within the deposit. The variance reduction factor is used for the affine variance correction and the indirect lognormal correction.

3.2 Affine Change of Support

The affine change of support model assumes that the shape of the histogram does not change and the variance is reduced by f. The reduction in variance is achieved by:

$$q' = \sqrt{f} \cdot (q - m) + m$$

where q is the quantile of the original value in the point scale distribution, m is the mean of the distribution, and q' is the quantile in the corrected block scale distribution.

The assumption that the shape of the histogram does not change is unrealistic. Because of this assumption, the affine correction introduces artificial minima and maxima and the shape of the resulting histogram is poor. The affine change of support model is only valid for small variance reductions, that is, $f \geq 0.7$.

Recoverable reserves can be calculated using the affine corrected distribution with the histogram based approach from Section 2.2.

3.3 Indirect Lognormal Change of Support

The indirect lognormal correction is the change of support model that should be used if the point and block distributions were lognormally distributed. It is more realistic than the affine correction as it assumes there is a change in the histogram shape.

The indirect lognormal correction needs to be applied in two steps. The first step in the correction is the variance reduction:

$$q' = aq^b \tag{17}$$

where q is the original quantile, q' is the variance corrected quantile, and

$$b = \sqrt{\frac{\ln(f \cdot CV^2 + 1)}{\ln(CV^2 + 1)}} \tag{18}$$

$$a = \frac{m}{\sqrt{f \cdot CV^2 + 1)}} \left[\frac{\sqrt{CV^2 + 1}}{m} \right]^b \tag{19}$$

The second step is to correct the mean of the block distribution back to the mean of the point distribution:

$$q'' = q' \frac{m}{m'} \tag{20}$$

where q'' is the final corrected quantile, m is the mean of the original distribution (also the target mean) and m' is the mean after the first correction.

The indirect lognormal correction is more realistic than the affine correction for lognormal distributions. It assumes an arbitrary shape change and does not impose artificial minima and maxima. Larger reductions in variance can be achieved with the indirect lognormal correction, $f \geq 0.5$.

3.4   Discrete Gaussian Model

The discrete Gaussian model (DGM), is used for change of support at different scales. It is a function defined by a polynomial expansion that needs to be fit to the data. Once the polynomials have been fitted, the function provides a mapping of the point variable Z to the Gaussian variable Y and vice-versa:

$$
\begin{aligned}
z(\mathbf{u}) &= \Phi(y(\mathbf{u})) \\
&\approx \sum_{n=0}^{np} \phi_n H_n[y(\mathbf{u})]
\end{aligned}
\tag{21}
$$

where np is the highest order term in the polynomial expansion, ϕ_n is a fitted coefficient for each term, and $H_n[Y(\mathbf{u})]$ is the hermite polynomial value defined by the term of the expansion and the y value. Equation (21) is referred to as the Gaussian anamorphosis.

Hermite polynomial's are defined by Rodrigues' Formula. The polynomial value can be calculated for any value of y and for any polynomial order n using the following formula:

$$
H_p(y) = \frac{1}{\sqrt{p!} \cdot g(y)} \cdot \frac{d^p g(y)}{dy^p}
\tag{22}
$$

For example, the first 6 Hermite polynomials are:

$$
\begin{aligned}
H_0(y) &= 1 & H_3(y) &= -\frac{1}{\sqrt{6}}(y^3 - 3y) \\
H_1(y) &= -y & H_4(y) &= \frac{1}{2\sqrt{6}}(y^4 - 6y^2 + 3) \\
H_2(y) &= \frac{1}{\sqrt{2}}(y^2 - 1) & H_5(y) &= -\frac{1}{2\sqrt{30}}(y^5 - 10y^3 + 15y)
\end{aligned}
$$

After the zero and first order polynomials have been calculated, the following recursive formula can be used to calculate the higher order polynomials, $H_p(y)$, when $p \geq 2$:

$$
H_{p+1}(y) = -\frac{1}{\sqrt{p+1}} y H_p(y) - \sqrt{\frac{p}{p+1}} H_{p-1}(y)
$$

We now need to calculate the ϕ coefficients to finish fitting the anamorphosis function. The first order coefficient is:

$$
\begin{aligned}
\phi_0 &= E\{\phi(Y(\mathbf{u}))\} \\
&= E\{Z(\mathbf{u})\}
\end{aligned}
\tag{23}
$$

or the expected value of $Z(\mathbf{u})$. Higher order ϕ coefficients can be calculated using:

$$
\begin{aligned}
\phi_p &= E\{Z(\mathbf{u}) \cdot H_p(Y(\mathbf{u}))\} \\
&= \int \phi(y(\mathbf{u})) \cdot H_p(y(\mathbf{u})) \cdot g(y(\mathbf{u})) \cdot dy(\mathbf{u})
\end{aligned}
\tag{24}
$$

The last expression can be approximated with the data at hand, as a finite summation:

$$
\phi_p \approx \sum_{\alpha=2}^{N} (z(\mathbf{u}_{\alpha-1}) - z(\mathbf{u}_\alpha)) \cdot \frac{1}{\sqrt{p}} H_{p-1}(y(\mathbf{u}_\alpha)) \cdot g(y(\mathbf{u}_\alpha))
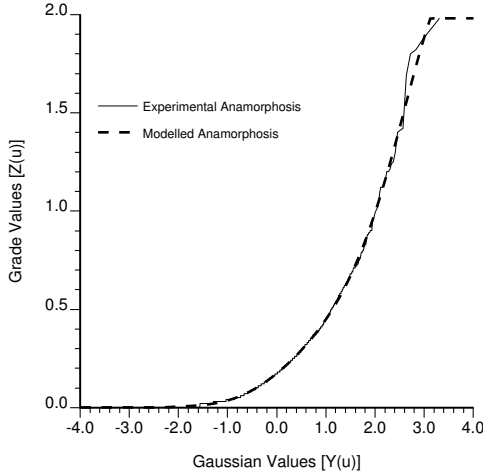\tag{25}
$$

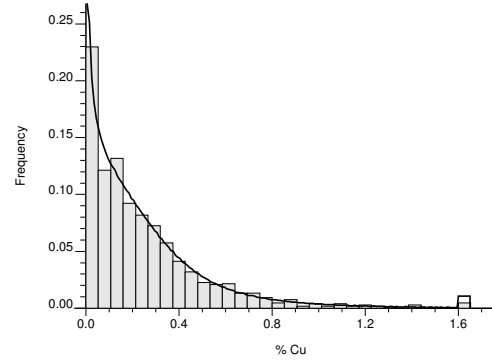**Figure 4:** Experimental and modeled gaussian anamorphosis.

Figure 5: Fitted Global Distribution.

The fitted ϕ coefficients must satisfy the following equality:

$$Var\left\{Z_{\mathbf{u}}\right\} \;\; = \;\; \sum_{n=1}^{np} \phi_n^2 \tag{26}$$

where $Var\left\{Z_{\mathbf{u}}\right\}$ is the variance of Z at the point support. If the summation is significantly different, the anamorphosis modeling should be checked.

There are two checks that can be done to determine the validity of the modeled anamorphosis function: (1) by comparing the Z to Y transformation function and (2) by comparing the global distribution from the data to the distribution from the anamorphosis. An example of the two plots are shown in Figures 4 and 5. The experimental anamorphosis is shown as a thin line with the modeled anamorphosis shown as a thick dashed line. The original grade distribution is shown as the bar histogram and the fitted distribution is shown as a line. The experimental and modeled anamorphosis functions should be identical, and the properties of the two distributions should be similar.

The discrete Gaussian model is used for estimating the change of support from point scale to block scale for a variable. The variability and shape of the block scale distribution is controlled by the anamorphosis.

This anamorphosis function (Equation 21) can be modified to account for the change of support from point data to block data by the addition of a change of support coefficient r:

$$\begin{aligned} Z(v) \;\; &= \;\; \Phi(Y(v)) \\ &\approx \;\; \sum_{n=0}^{np} r^n \phi_n H_n\left[Y(v)\right] \end{aligned}$$

By calculating the value of r, we can determine the distribution of grades for volumes of support larger than the point samples.

The calculation of r requires the variance of the larger support volumes. Typically, there is not enough data available to do this explicitly. The variance of the larger blocks can be

estimated using the modeled variogram of the point data. Dispersion variance theory gives us the following relation:
$$\sigma_v^2 = \sigma_{\mathbf{u}}^2 - \bar{\gamma}_{v,v}$$
where v is the SMU support volume and $\mathbf{u}$ is the point support volume. Therefore we can calculate the variance of the larger blocks using the modeled variogram. Recall Equation 26:

$$Var\left\{Z_{\mathbf{u}}\right\} \quad = \quad \sum_{n=1}^{np} \phi_n^2$$

Since this equality must be true for the point support, it must also be true for the block support. By including r in the equality, the equation becomes:

$$
\begin{aligned}
Var\left\{Z_v\right\} \quad &= \quad \sigma_v^2 \\
&= \quad \sigma_{\mathbf{u}}^2 - \bar{\gamma}_{v,v} \\
&= \quad \sum_{n=1}^{np} r^{2n} \phi_n^2
\end{aligned}
\tag{27}
$$

where $Var\left\{Z_v\right\}$ is the variance of Z at the SMU support. The only unknown parameter is r. Any optimization method can be employed to find the value of r. A bisector search method is used in the DGM program written at CCG.

After calculating the change of support coefficient, the block scale distribution can be estimated. Global reserves are then estimated using the upscaled block distribution. The discrete Gaussian model is a fairly robust upscaling algorithm. It makes the assumption that the distribution tends to a Gaussian shape as the variance at the block scale decreases.

4   Local Resource Estimation

Although global resource estimation techniques can be used at initial mine evaluation stages, they cannot be used when considering local areas in the mine. Pit optimization and short/long term planning require local estimation of the grades. Global reserves can be calculated using the local estimates and compared with the global reserves calculated previously.

Local resource or reserves are calculated using an estimation technique, such as ordinary kriging, or a simulation technique, such as sequential Gaussian simulation. The estimates are made directly, by kriging or simulating blocks that will become the SMU's at the time of mining, or indirectly, by estimating larger planning blocks and then estimating the reserves within the larger mining panels. The different methods and implementation suggestions are presented below.

4.1   Kriging

Kriging can be used to estimate the grade of the SMU sized blocks directly. These blocks are used for classification and reserve calculation. It is a fast way to get recoverable reserves. However, when the drill hole data are widely spaced relative to the blocks being kriging

significant smoothing can be introduced. This large degree of smoothing will make the calculated reserves unreliable [3].

There are many flavors of kriging available: (1) simple kriging, (2) ordinary kriging, (3) multiGaussian kriging, (4), disjunctive kriging, (5) lognormal kriging, and (6) indicator kriging.

Simple and ordinary kriging work with the data in its original units. The estimates are a linear combination of the data using the modeled variogram to calculate the weights. As mentioned earlier, kriging smoothes the estimates and can introduce a bias when estimating blocks that are small relative to the data spacing.

Krige and Assibey-Bonsu have proposed a correction for simple kriging that allows the direct estimation of SMU sized blocks that corrects for the smoothing effect of kriging [1]. The case study presented in their paper shows that the correction allows an unbiased estimation of the reserves.

Multi-Gaussian kriging uses the normal score transformed values of the data. The block estimates are calculated in Gaussian space and then need to be back transformed. An advantage of this method is that the conditional distributions for each block are known.

Disjunctive kriging describes a flavour of kriging that uses simple kriging to estimate the Hermite polynomial values at unsampled locations [7]. The estimate is given as a linear combination of these estimated values, and weighted by the coefficients of fitting the global distribution [6]. This method is quite flexible, allowing for use of different isofactorial models to be used; however, the lack of simple programs has hindered its implementation in practice.

Lognormal kriging works with the log transformed values of the original data, and has found application when the original data are positive valued and especially if it is positively skewed [11]. A logarithmic transform is applied to the data to normalize it, any kriging method can then be applied to generate estimates in normal scores. This seems straightforward, however, the problem arises in the back transformation when any errors in the estimate or estimation variance can be exponentially magnified [11]. Further, deviations of the original data from lognormality may invalidate its use.

The block scale CDF can be estimated using indicator kriging. Multiple cutoffs are applied to the data and the indicators are kriged. Each block requires p kriging equations to be solved, where p is the number of cutoffs. The resulting estimates define the block scale CDF in original units. Median indicator kriging is a subset of indicator kriging. Indicator kriging requires a separate variogram for each cutoff, while median indicator kriging assumes that each cutoff has the same variogram. The assumption may or may not be valid depending on the data.

4.2   Uniform Conditioning

Recall that kriging small blocks in relation to the data spacing can be problematic. However, we can krige larger blocks, called panels, to reduce the possibility of biased estimates. Uniform conditioning estimates the distribution of the SMU's within each estimated panel. In other words, given that we know the grade of a large block, the panel that has been estimated, we can calculate the distribution of the SMU's that are in the panel. Recoverable

reserves are calculated using the SMU distribution that has been derived from the panel estimate and the change of support model.

A change of support model is needed to estimate the SMU block distribution within each panel. Uniform conditioning uses the discrete Gaussian model to accomplish the change of support.

There are a few limitations to the uniform conditioning method. One is that it does not provide any spatial location for the SMU's contained in each panel. The high grade SMU's and the low grade SMU's could be anywhere. The second limitation is that two panels with the same estimated grade will have the same reserves irrespective of surrounding data.

4.3   Conditional Simulation

All of the local reserve estimation techniques presented so far provide an estimate of the reserves. They do not provide a *distribution* of uncertainty in the estimate. Conditional simulation allows us to generate multiple realizations to assess both global and local uncertainty in our model.

The very first simulation techniques generated unconditional Gaussian simulations. Such methods include turning bands simulation, moving average techniques and LU simulation. The unconditional Gaussian simulations then needed to be conditioned to the data using kriging. This method is theoretically correct but it involves 2 distinct steps.

Modern simulation methods do not need to be performed in 2 steps. They are capable of simultaneously generating a simulated realization and conditioning to the data. The most common simulation algorithm currently is sequential Gaussian simulation. Working within the Gaussian paradigm first requires a Gaussian transformation of the data. The transformation, if done incorrectly, can impact the results of the simulation. Indicator simulation infers the shape of distribution and draws stochastically from it, but this requires a non-linear transform of the data to indicator variables at different thresholds. Too few thresholds results in a poorly inferred shape, while too many thresholds requires more professional time and effort in inference of variogram parameters. Direct sequential simulation is a relatively new algorithm under development that does not require data transformation.

4.4   Upscaling Local Resources for Global Resources

Aside from a local distribution of uncertainty, the move towards simulation from traditional estimation methods allows one other advantage. A realization of simulated values can be directly upscaled to any arbitrary volume and the change in resources/reserves evaluated directly. No change of support model is required, hence no distributional assumptions are imposed.

Upscaling in this instance generally involves taking the simulated grid and averaging all relevant blocks within the volume of interest. For instance, if the panel volume represents a monthly production period, only those blocks within the production area need to be considered and averaged. As before, the panel volume need not be regular, polygons may be considered that may or may not be contiguous regions within the deposit representing multiple advancing faces in production. This averaging of multiple simulated grades yields the panel grade. Repeating this averaging process over multiple realizations yields a dis-

tribution of uncertainty in the panel grade; thus, the variance of this panel grade can be directly obtained.

The extension of this process for increasing volumes up to the deposit size is straightforward, making global resource estimation using simulated realizations quite simple. In practice, the biggest limitation to this approach is the computational effort to generate and store these multiple realizations; however, as computers become faster and storage becomes less of an issue, it is only a matter of time before these issues are resolved.

5   Case Study

A full case study was completed using a data set provided by Placer Dome Technical Services. The particulars of the mine will be kept confidential.

Four methods were used to calculate recoverable reserves for the case study: (1) change of support models, (2) ordinary kriging, (3) uniform conditioning, and (4) simulation. The results from each method will be used to calculate a grade tonnage curve. All of the grade tonnage curves will be compared to see if any methods are substantially different than average.

The SMU size for the study is 10x10x10m, and the panel size, for uniform conditioning, is 60x60x20m. The following steps were done as part of the reserve estimation:

- exploratory data analysis,

- spatial continuity analysis,

- direct estimation of the global resources,

- kriging for global resources,

- uniform conditioning for global resources,

- conditional simulation for global resources, and

- comparison of the results.

The results from each step are documented below and issues discussed.

5.1   Data

The topography model of the existing mine was provided by Placer Dome. Although this information will not be used for this project, it was used to get a better understanding of the mining operation. The topography is shown in Figure 6.

The composited drillhole data contains the x, y, and z locations of each composite along with the gold and silver grades. This was imported into GOCAD for visualization purposes (Figure 7). Figures 8 and 9 show the drillhole data with the mine topography. Figure 10 shows a plan view of the entire drill hole data set. Very litte information can be gleaned from this figure. Figure 11 shows the same plan view, but only the samples that are at an elevation of 200m. It is easy to see on the graph that the higher grade areas are located towards the middle of the deposit with the lower grades at the extremities. Figure 12 shows a vertical cross section of the deposit.
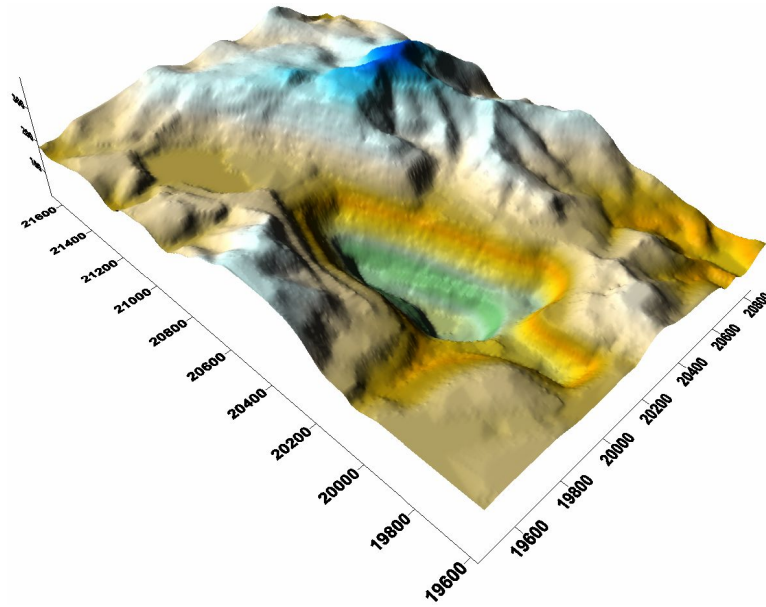
Figure 6: Topography of the existing mine.
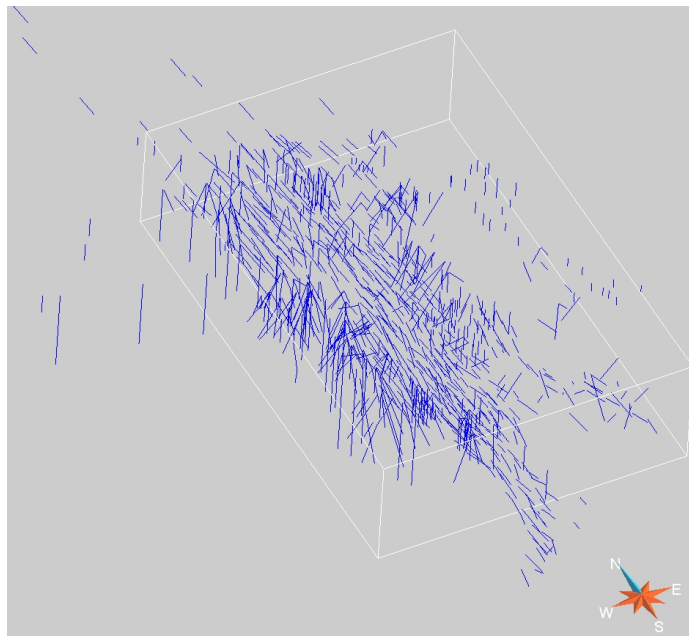
Figure 7: Drillhole data set.
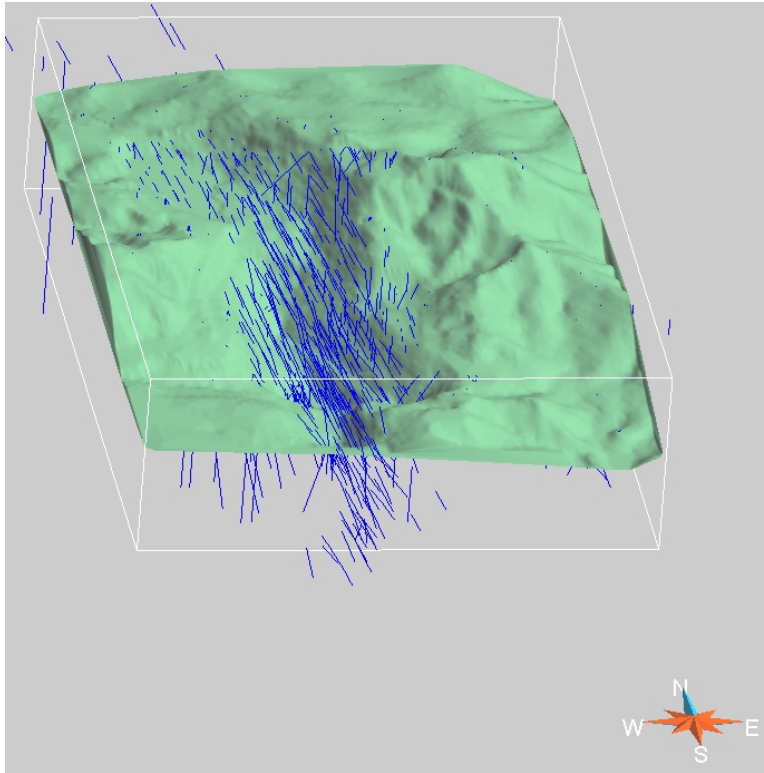
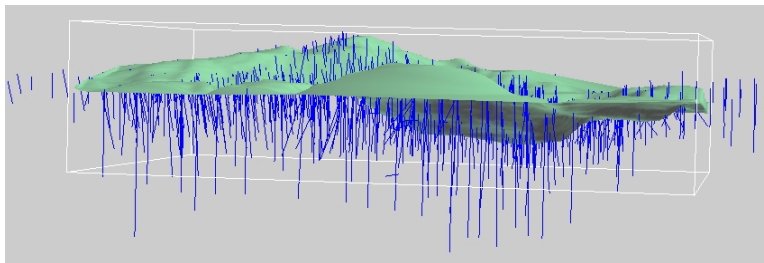**Figure 8:** Topography and drill holes.

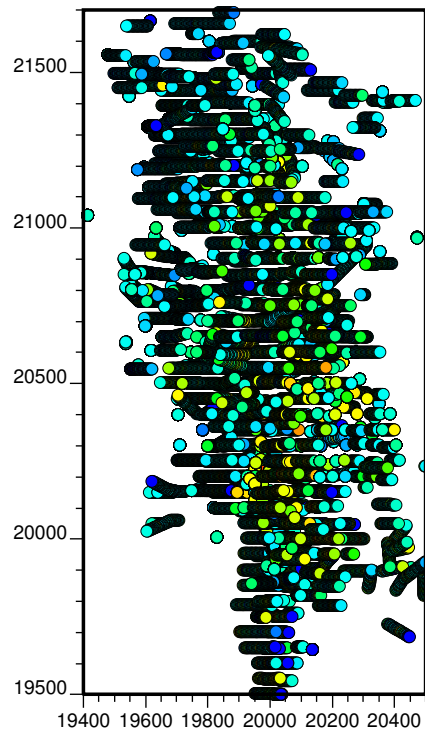**Figure 9:** Topography and drill holes.

Figure 10: Plan view of all the drill holes.
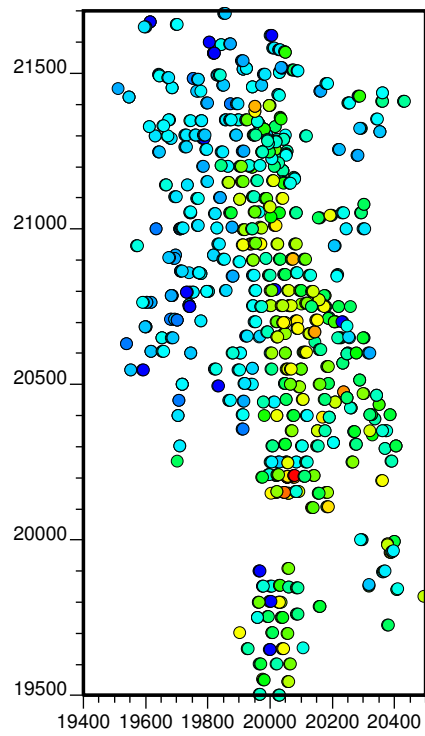
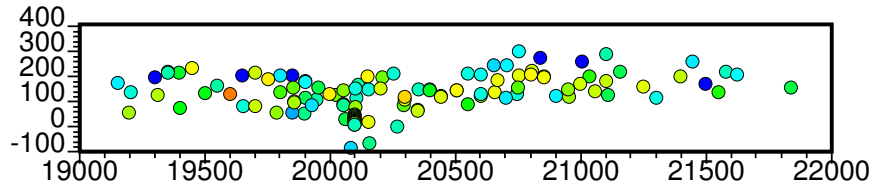

Figure 11: Plan view of bench 200.

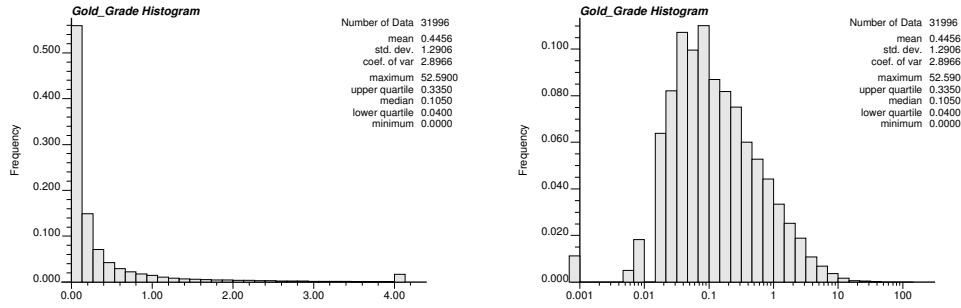**Figure 12:** Vertical cross-section looking west at 2000 meters East.

Figure 13: Gold grade histograms.

5.2 Exploratory Data Analysis

The goal of the exploratory data analysis is to get representative statistics for the variables in the deposit. The important mineral for this study is gold.

The average gold grade is 0.44 with a variance of 1.66. Figure 13 shows the gold histogram. The histogram on the left has an arithmetic axis, while the histogram on the right has a lognormal axis. The latter plot shows that the gold distribution is close to being lognormal.

Samples are almost always collected in a spatially biased fashion. Declustering is used to determine statistics that are representative for the entire deposit from the biased samples. A declustering cell size of 150m was chosen using the plots in Figure 14 and the average data spacing in the sparsely sampled areas. The declustering results are shown in Figure 15. As expected, the mean and variance for the gold and silver has decreased.

5.3 Normal Score Transformation

Sequential Gaussian simulation and the discrete Gaussian model for change of support require the data to be standard normal. This means the data must follow a normal distribution with a mean of zero and a variance of one. The gold grades were twice transformed to normal scores. The first time the declustering weights were not used. These normal score values will be used for calculating variograms. The declustering weights were used for the second transformation. These normal score values will be used for the conditional simulation. Figure 16 shows the normal score gold grades.
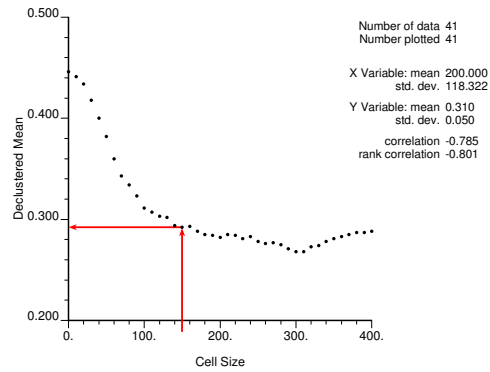
Figure 14: Declustered mean versus cell size.
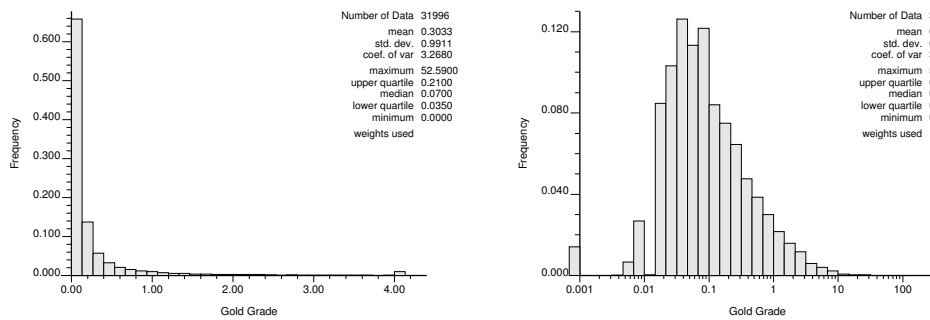
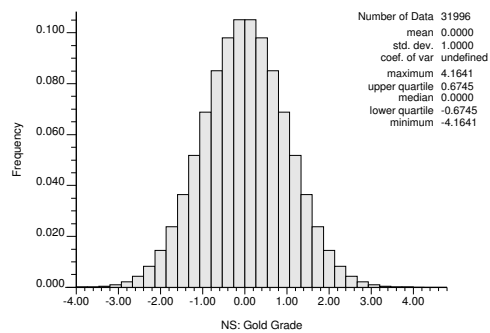**Figure 15:** Declustered gold histogram.

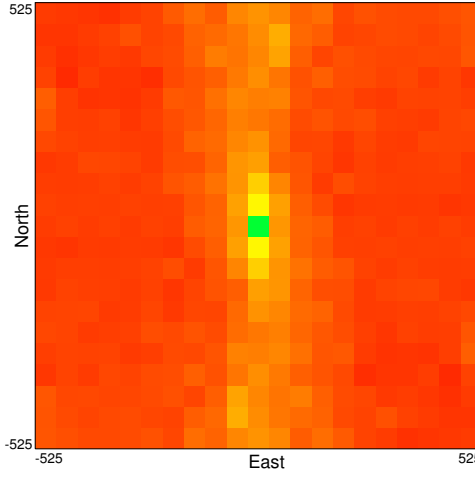**Figure 16:** Normal score transformed gold histogram.

Figure 17: Real space variogram maps.

5.4   Variogram Analysis

The real space experimental variograms showed pure noise; changes to the the lag spacing, tolerance, or minimum number of pairs made little difference to ease model inference. Figure 17 shows the poor quality of the real space variogram maps. The spatial continuity is much easier to see in the normal score variogram maps (Figure 18). The major direction of continuity was chosen as -10° azimuth from north, the minor direction is 80° azimuth from north and the vertical variogram is straight down.

Since the calculated variograms for the original gold variable were pure nugget, some alternate measures of spatial continuity were evaluated. Specifically, correlograms, pairwise relative and the covariance were also analyzed; using a combination of these measures, the following real space gold variogram was obtained (see Figure 19):

$$\gamma\left(\mathbf{h}\right) = 0.25 + 1.12 \cdot \exp_{\substack{\text{hmax=85}\\\text{hmin=60}\\\text{hvert=17}}}\left(\mathbf{h}\right) + 0.3 \cdot \text{sph}_{\substack{\text{hmax=440}\\\text{hmin=120}\\\text{hvert=250}}}\left(\mathbf{h}\right)$$

The normal score variograms were much better behaved than the real space variograms. There was no need to consider a robust variogram measure. The experimental normal score variogram is shown in Figure 20. The normal score gold variogram model is:

$$\gamma\left(\mathbf{h}\right) = 0.15 + 0.52 \cdot \exp_{\substack{\text{hmax=60}\\\text{hmin=55}\\\text{hvert=90}}}\left(\mathbf{h}\right) + 0.3 \cdot \text{sph}_{\substack{\text{hmax=1300}\\\text{hmin=350}\\\text{hvert=270}}}\left(\mathbf{h}\right)$$

5.5   Direct Estimation of the Global Resources

The variance of blocks within the deposit was estimated using the modeled real space variograms. Table 1 contains the gammabar value for the SMU sized blocks, the estimated variance of the blocks within the deposit, and the variance reduction factor for the change of support calculation.
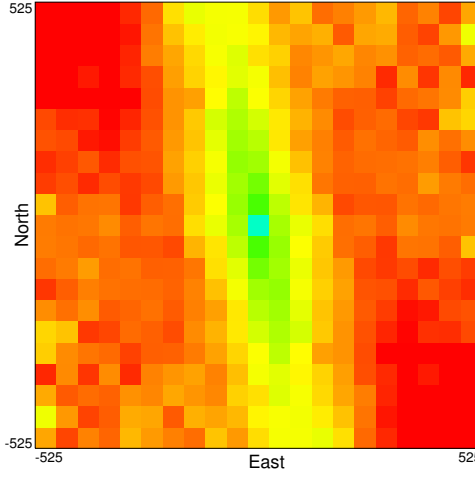
Figure 18: Normal space variogram maps.

Table 1: Dispersion variances.

	$\gamma(v, v)$	$\sigma^2(v, A)$	f
SMU	0.452	0.530	0.540

These values were used to estimate the block scale distribution through the 3 change of support models previously documented. The results of the affine change of support are shown in Figure 21, the results for the indirect lognormal change of support are shown in Figure 22, and the results for the discrete Gaussian model are shown in Figure 23. All of the figures have the original gold grade histogram on the left and the volume corrected histogram on the right. The indirect lognormal and the discrete Gaussian histograms look good, while the affine corrected histogram does not look very good. This is because the affine change of support assumes that the histogram shape does not change.

5.6 Kriging for Global Resources

Ordinary kriging was used to estimate the grade for the SMUs directly. This was not expected to give good results but was done for comparison purposes. Figure 24 shows the kriged SMU gold grades. The kriged panel estimates are shown in Figure 25. The panel estimates will be used for the uniform conditioning.

5.7 Uniform Conditioning for Global Resources

The panel estimates were used to calculate reserves with uniform conditioning. The reserves were calculated for each panel and the results upscaled to provide global reserves.

5.8 Conditional Simulation for Global Resources

Sequential Gaussian simulation was used to build conditional simulations of the gold grade. The first step was to generate 51 realizations of the gold grade at a fine scale, and then
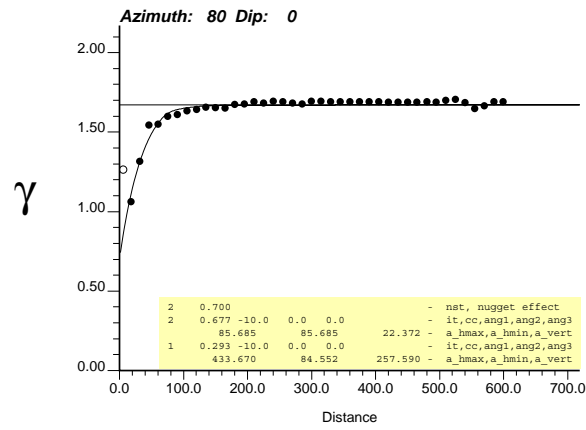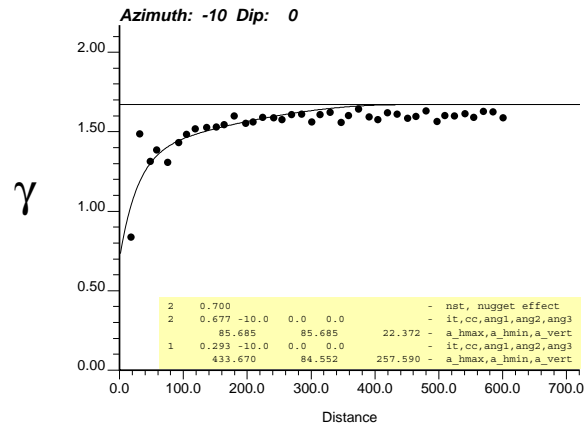
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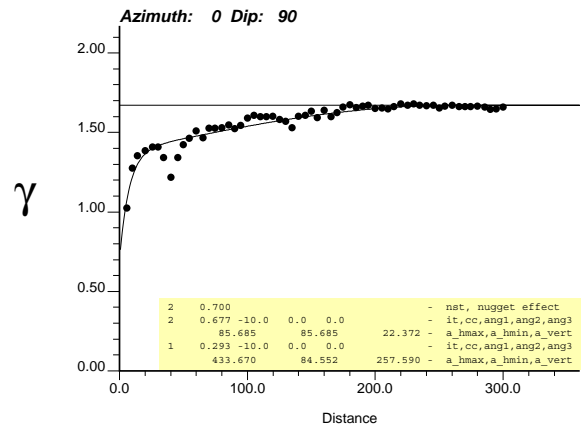
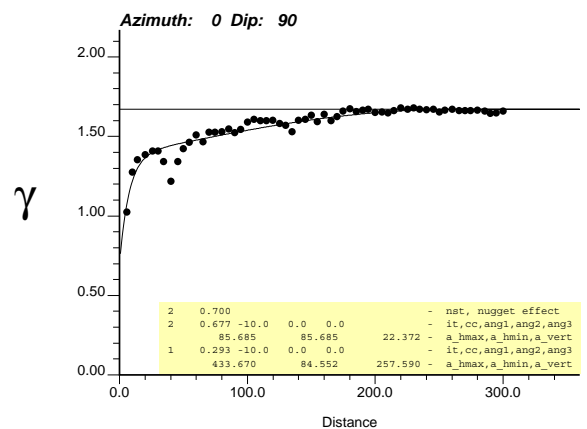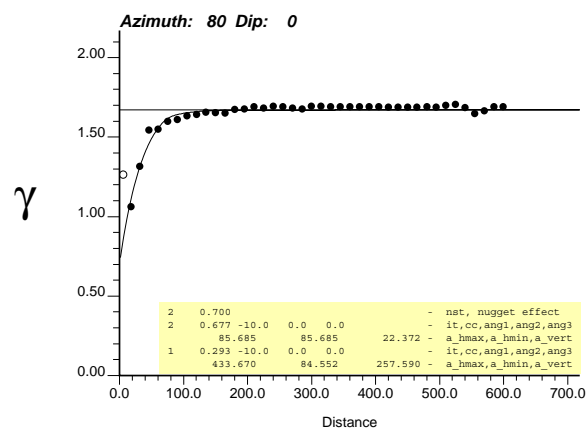**Figure 19:** Real space modeled gold variogram.

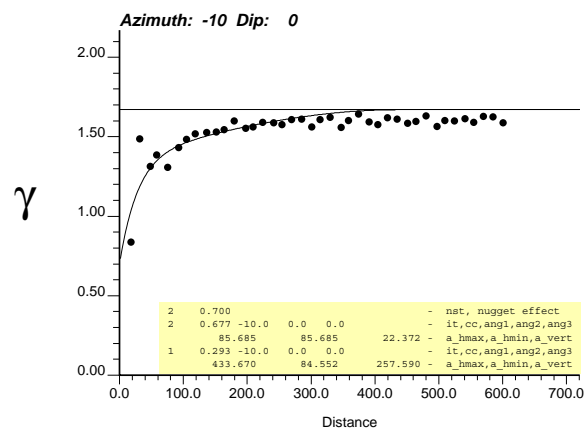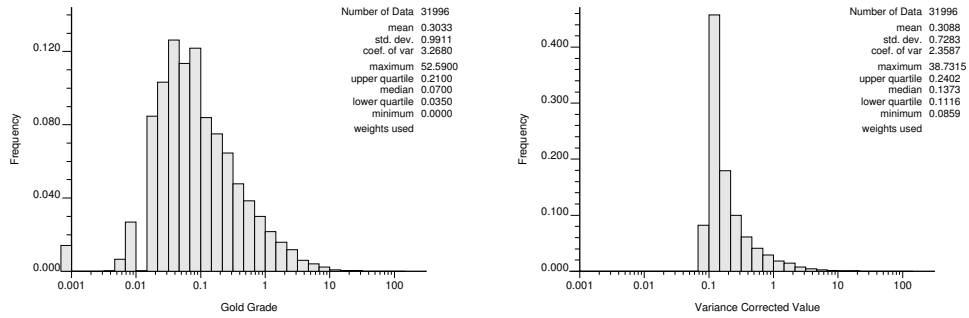**Figure 20:** Normal score modeled gold variogram.
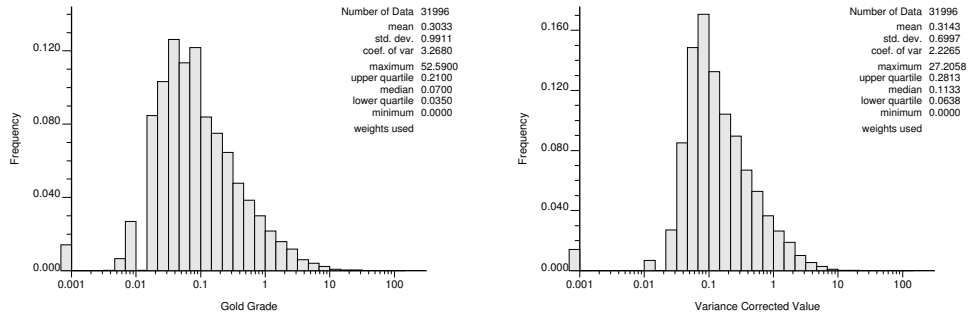
Figure 21: Affine corrected histogram.
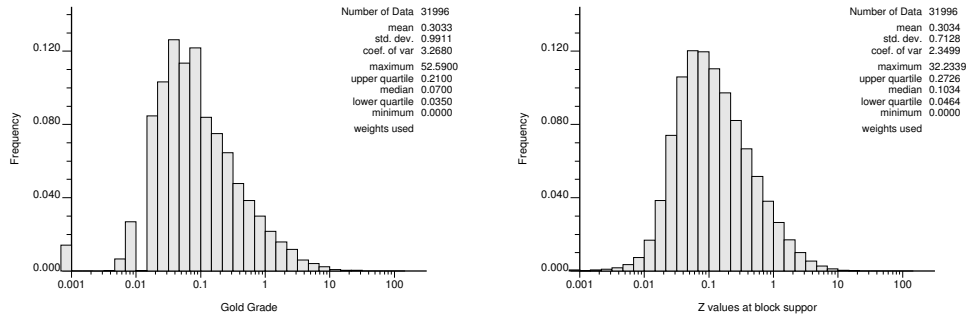

Figure 22: Indirect lognormal corrected histograms.

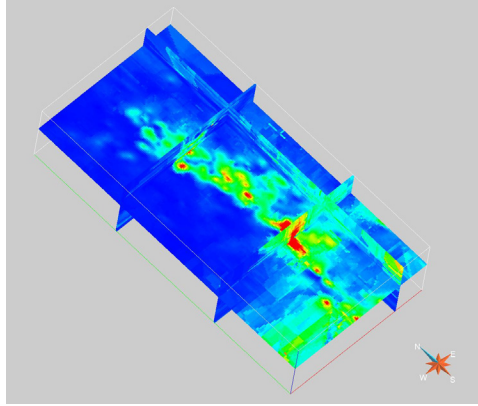**Figure 23:** Discrete Gaussian model corrected histograms.

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Figure 24: Ordinary kriging of the SMU's directly.
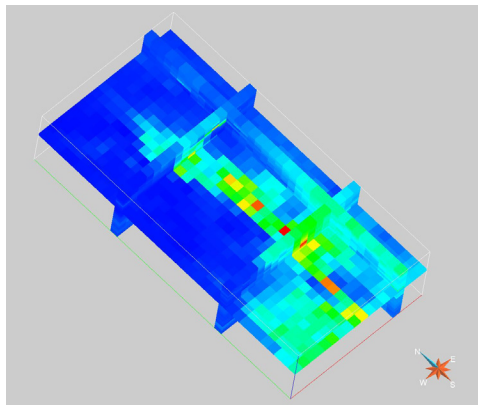

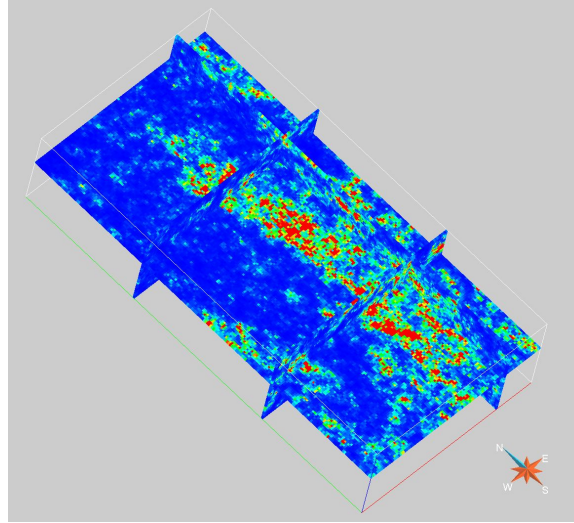**Figure 25:** Ordinary kriging of the panel grades for uniform conditioning.

Figure 26: Back transformed simulation, block averaged to the SMU blocks.

upscale the results to the SMU blocks. Simulating to a very small block and then block averaging up to the larger SMU sized blocks allows for the change of support to be accounted for directly. Figure 26 shows one realization of the block averaged gold grades.

5.9   Comparison of the Results

Now we can calculate and compare the grade tonnage curves for the different methods. The grade tonnage curves for gold are shown in Figure 27. The solid black line shows the average grade tonnage curve for the simulation results, the dashed black line shows the 90% probability interval (P05-P95) from simulation and the other coloured lines represent the other methods. A number of important observations can be made: (1) the simulation results match very closely with the uniform conditioning results, (2) the discrete Gaussian and indirect lognormal change of support models are very similar to the simulation results, and (3) the affine change of support and the ordinary kriging results are poor. The change of support models are dependent on the parameters that define the change in variance, and thus can change dramatically with a small change in the variogram. In other words, the change of support models could be made to match the simulated results even better with revised change of support parameters. Figure 28 is a close up of the grade tonnage curve.

For interest sake, the e-type from the simulation results (that is the local average over all realizations) were also considered and the corresponding grade tonnage curve calculated. The grade tonnage curve is shown in Figure 29. Note that the e-type grade tonnage curve is very similar to the kriging curve. It does not provide a good estimate of the recoverable resources. Rather than averaging the grades and then calculating the grade tonnage curve (as done here), one should calculate the grade tonnage curve associated to all realizations, and then take the average of the grade tonnage curves (as illustrated by the black line on the plots).
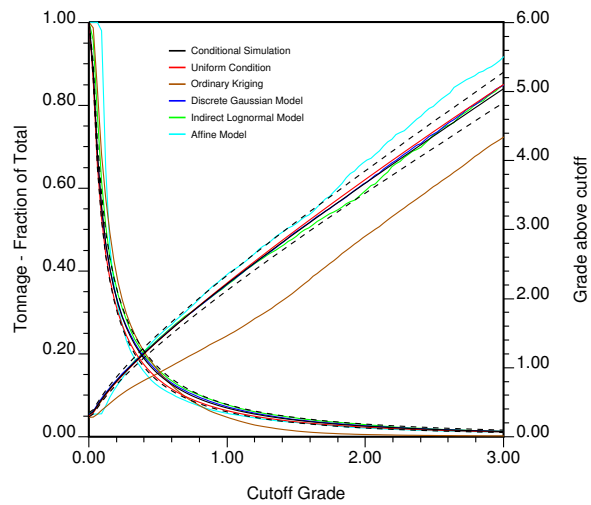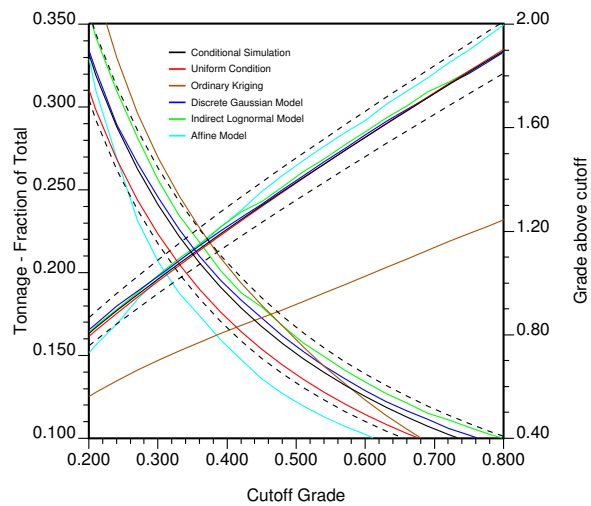
Figure 27: Grade tonnage curve.

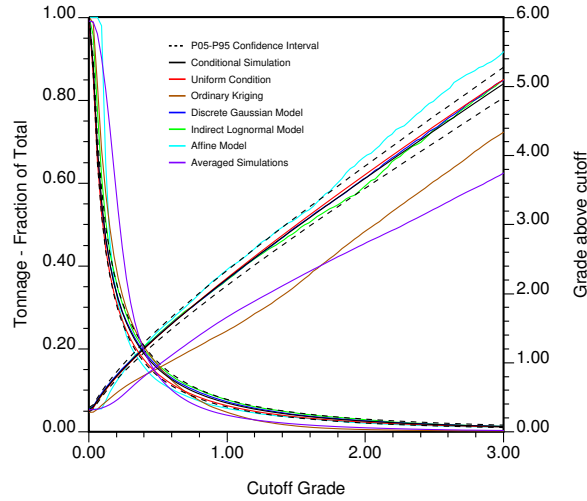**Figure 28:** Close up of the grade tonnage curve.

Figure 29: Average simulation grade tonnage curve or average grade tonnage curve.

6 Discussion and Conclusions

Recoverable reserves are vital during project development stages. Mining companies will invest millions of dollars to develop a mine using an estimate of the recoverable reserves. Over or under estimating the reserves can result in the a huge loss to the mining company. To make the process harder, there are many different methods for calculating recoverable reserves from which to choose.

Data based methods use the data at hand and a change of support model to estimate the SMU block distribution over the entire deposit. Recoverable reserves can then be calculated using the upscaled histogram. The affine change of support model makes an unreasonable assumption that the histogram will not change shape as the support volume increases. For this reason, the affine model is not recommended. The indirect lognormal model assumes that the point scale distribution and the block scale distribution are lognormal. The shape of the histogram does change as the support volume increases. The indirect lognormal model will produce acceptable results when the data are approximately lognormal. The discrete Gaussian model works on the normal score transform of the data and a fitted polynomial to perform the change of support. The discrete Gaussian model can be fit to almost any input distribution, but it makes the assumption that the distribution tends to Gaussian as the block size increases. It also produced good results.

Ordinary kriging of the SMU blocks did not produce good results. The smoothing effect of kriging, and estimating small blocks in relation to the data spacing produced a biased estimate of the recoverable reserves.

Ordinary kriging of the larger panels and then using uniform conditioning produced good results. The panel estimates are more robust than the smaller SMU estimates and uniform conditioning correctly estimated the SMU distribution within each panel. However, uniform conditioning does not provide a spatial location for the SMU sized blocks. This makes it difficult to do a proper mine plan or pit optimization compared to a block model at the SMU scale.

Sequential Gaussian simulation produced good results. The benefit of simulation is the ability to assess local and global uncertainty. The simulation results matched very closely to the change of support models and uniform conditioning. The downside of simulation is that it takes more time to generate and postprocess the realizations compared to the histogram based methods, or uniform conditioning.

Averaging, or the expected value, is useful for presenting a summary of a full scale simulation study. However, care must be taken to ensure that the expected value is taken at the correct moment. The expected value must be taken after any non-linear transformation or transfer functions. For recoverable reserves, that means the expected value of the realizations should not be calculated before the grade tonnage curve, the expected value should be taken from the multiple grade tonnage curves calculated from the multiple realizations.

Direct simulation methods presents some interesting possibilities for reserve estimation. The avoidance of data transformation should make it easier to interpret, while the added benefit of directly simulating any arbitrary volume can mitigate the computationally expensive post-processing of multiple realizations. There are working versions of direct sequential simulation available. Future work in this area should include a comparison of DSS with conventional reserve estimation techniques.

7   Acknowledgements

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